3 Building Abstractions with Procedures

What's in This Set of Notes ?

- Conditional Expressions and Predicates
- Procedures and The Processes They Generate
- Other Forms of Recursion
- Procedures
- Summary

3.1 Conditional Expressions and Predicates

- Following construct is called case analysis

```
|x| =     x if x>0
          0 of x = 0
         -x if x <= 0
```

- Requires **special form** called **cond**

```
(cond  ( <p1> <e1>)
       ( <p2> <e2>)
       ...
       ( <pn> <en>))
```

- Expression of the form (<p> <e>) is called a clause
- First expression in clause is called a predicate, e.g. whose values is either true or false

```
(define (abs x)
   (cond   (( > x 0) x)
           (( = x 0) 0)
           ((< x 0) (- x))))
```

- Another way

```
(define (abs x)
   (cond   (( < x 0) (- x))
           (else x)))
```

- Another way

```
(define (abs x)
      (if (< x 0)
          (- x)
          x))
```

- Requires **special form** called if

```
(if  <predicate> <consequent> <alternative> )
```

- Primitive Predicates: <, = , >
- Logical Composition Operations

(and <e1> ... <en>) evaluate left to right, as soon as one expression is false return false

(or <e1> ... <en>) evaluate left to right, as soon as one expression is true return true

```
(not <e>)
```

```
(define (>= x y)
    (or ( > x y) (= x y))
```

```
(define (>= x y)
   (not (< x y))
```

Example: Compute Square Roots using Newton's method of successive approximations

1. guess y for value of square root of number x
2. to get a better guess average y with x/y

For square root of 2

Guess (y)	Quotient (x/y)	Average ((x/y) + y) / 2 => gives next guess
1	2/1 = 2	(2 + 1)/2 = 1.5
1.5	2/1.5= 1.333	(1.3333 + 1.5) / 2 = 1.4167
1.4167	2/1.4167= 1.4118	(1.4167 + 1.4118) / 2 = 1.4142
1.4142	...	...

```
(define (good-enough? guess x)
   (< (abs (- (square guess) x)) 0.001))
```

```
(define (average x y)
   (/ (+ x y) 2))
```

```
(define (improve guess x)
   (average guess ( / x guess)))

(define (sqrt-iteration guess x)
   (if (good-enough? guess x)
         guess
         (sqrt-iteration (improve guess x) x)))

(define (sqrt x)
   (sqrt-iteration 1.0 x))
```

- Sqrt is first example of process defined by a set of mutually defined procedures
- Problem breaks up naturally into subproblems: decomposition strategy
- Details of how sqrt is computed can be hidden (black box) creating a so-called procedural abstraction
- Take the procedure square, we know what we want, but how does it do it?

```
(define (square x) ( * x x))

(define (square x) (exp (double (log x))))

(define (double x) ( + x x ))
```

- A procedure definition should be able to supress details
- Users of procedure may not have written it
- **Users should not need to know how a procedure is implemented in order to use it!**

Local names

- Formal parameter of procedure has special role, it doesn't matter what name of the formal parameter has
  - such a name is called a **bound variable,** the procedure definition **binds** its formal parameters
  - the meaning of the procedure definition is unchanged if the name of bound variable is consistently renamed
- If variable not bound, is **free**
- The set of expressions for which a binding defines a name is called the **scope** of the name

---

- How to hide choice of procedure names from users of sqrt?
- Allow procedure to have internal definitions local to procedure
- Such nesting of definitions is called **block structure**

Developers view of sqrt procedure

```
(define (sqrt x)

  (define (good-enough? guess)
      (< (abs (- (square guess) x)) 0.001))

  (define (improve guess)
      (average guess ( / x guess)))

  (define (sqrt-iteration guess)
      (if (good-enough? guess)
        guess
        (sqrt-iteration (improve guess))))

  (sqrt-iteration 1.0))
```

Users view of sqrt procedure

```
(define (sqrt x) ....)
```

User cannot directly use procedures good-enough?, improve and sqrt-iteration

- **Note**, x is now a free variable. x gets its value from the argument with which the enclosing procedure sqrt is called
- This is called lexical scoping

3.2 Procedures and the Processes They Generated

- A procedure is a pattern for the *local evolution* of a computational process
- What to make statements about the overall/global behavior of the process (*difficult*)
- To become experts, we must learn to *visualize* the processes generated by various types of procedures.

Factorial function

n! = n * (n - 1) * (n - 2) . . . 3 * 2 * 1
n! = n * (n - 1)!

Linear Recursive Process

```
(define (factorial n)
   (if ( = n 1)
      1
      (* n (factorial ( - n 1)))))
```

Substitution Model for (factorial 5)

```
(factorial 5)
(* 5 (factorial 4))
(* 5 (* 4 (factorial 3)))
(* 5 (* 4 (* 3 (factorial 2))))
(* 5 (* 4 (* 3 (* 2 (factorial 1)))))
(* 5 (* 4 (* 3 (* 2 1))))
(* 5 (* 4 (* 3 2)))
(* 5 (* 4 6))
(* 5 24)
120
```

- This is a linear recursive process
  - expansion followed by contraction
  - chain of deferred operations (*)
  - interpreter keeps track of operations to perform later
  - amount of information to keep track of grows linearly

Linear Iterative Process

```
(define (factorial n)
   (factorial-iteration 1 1 n))

(define (factorial-iteration product counter
maximum)
   (if (> counter maximum)
      product
      (factorial-iteration (* counter product)
(+ counter 1) maximum)))
```

Substitution Model for (factorial 5)

```
(factorial 5)
(factorial-iteration 1 1 5)
(factorial-iteration 1 2 5)
(factorial-iteration 2 3 5)
(factorial-iteration 6 4 5)
(factorial-iteration 24 5 5)
```

(factorial-iteration 120 6 5)
120

- This is a linear iterative process
  - state can be summarized by a fixed number of state variables together with a rule on how to update them
  - end test is optional
  - number of steps grows linearly

Comment

- The above **procedures** are recursive (the procedure syntactically refers to itself)
- Don't confuse this with the notion of a recursive **process** (how computation evolves)
- Ada, Pascal and C are designed in such a way that recursive procedures consume an amount of memory that grows with the number of procedure calls
- Ada, Pascal and C therefore require special looping constructs (repeat-until, while, for)
- **Scheme executes an iterative process in constant space (this property is called tail-recursive)**

3.3 Other Forms of Recursion

Tree Recursion

Fibonacci Numbers

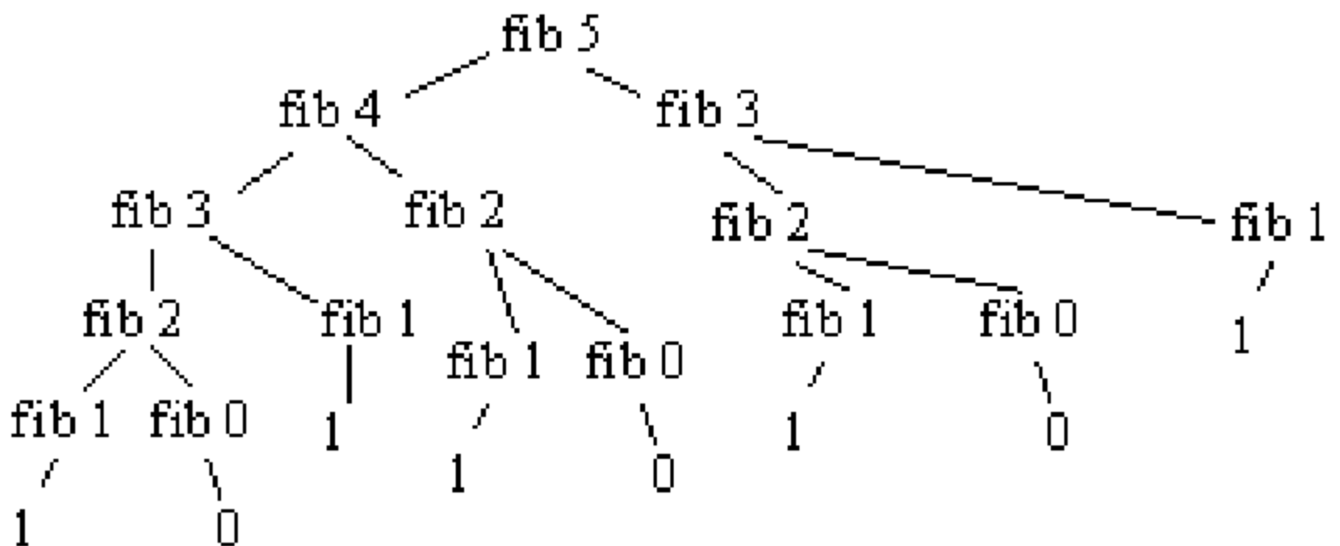
Fib(n) = 0	if n = 0
Fib(n) = 1	if n = 1
Fib(n) = Fib(n - 1) + Fib(n - 2)	otherwise

0, 1, 1, 2, 3, 5, 8, 13, 21 ....

Recursive Process

```
(define (fib n)
   (cond (( = n 0) 0)
            (( = n 1) 1)
            (else
               (+ (fib (- n 1))
```

```
(fib ( - n 2))))))
```



- process looks like a tree
- inefficient due to duplicate computation

Iterative Process

```
(define (fib n)
   (fib-iteration 1 0 n))

(define (fib-iteration a b count)
   (if (= count 0)
         b
         (fib-iteration ( + a b) a (- count 1))))

(fib 4)
(fib-iteration 1 0 4)
(fib-iteration 1 1 3)
(fib-iteration 2 1  2)
(fib-iteration 3 2  1)
(fib-iteration 5 3  0)
3
```

3.4 Procedures

- We have seen that ***procedures*** are abstractions that describe compound operations on numbers independent of the particular numbers

- Used to express general methods of computation independent of functions
- Using only numerical processing we will severely limited our ability to create abstractions
- Want to be able to do the following:
  - build higher order procedures
  - assign names to them
  - work with them

Procedures as Arguments

- Compute the sum of integers from a through b

```
(define (sum-int a b)
   (if (> a b)
      0
      (+    a
            (sum-int (+ a 1) b))))
```

- Compute the sum of cubes from a through b

```
(define (cube x) (* x x x ))

(define (sum-cubes a b)
      (if ( > a b)
         0
         (+ (cube a)
            (sum-cubes (+ a 1) b))))
```

- Compute (1/(1 * 3)) + (1/(5 * 7)) + (1/(9 * 11)) + . . .
converges to pi/8

```
(define (pi-sum  a b)
      (if ( > a b)
         0
         (+  (/ 1.0 (* a (+ a 2))
               (pi-sum (+ a 4) b))))
```

- **There is a pattern here:**

(define (**<name>** a b)
   (if (> a b)
      0
      (+ (**<term>** a)
         (**<name>** (**<next>** a) b))))

(define (sum **term** a **next** b)

```
  (if ( > a b)
     0
     (+ (term a)
        (sum term (next a) next b ))))
```

- Let's redo our sum-int and sub-cubes example
- First we must define term and next functions for our examples

```
(define (inc n) (+ n 1))
(define (identity n) n)

(define (sum-int a b)
   (sum identity a inc b))

(define (sum-cubes a b)
   (sum cube a inc b))

(define (pi-sum a b)
   (define (pi-term x) (/ 1.0 (* x (+ x 2))))
   (define (pi-next x) ( + x 4))
   (sum pi-term a pi-next b))
```

Special form Lambda

- Want to be able to create unnamed procedures
- More convenient than defining trivial procedures such as pi-term and pi-next
- Use special form lambda

```
(lambda (<formal parameters>) <body>)
```

(lambda (x) (+ x 4)) returns a procedure that increments x by 4 like pi-next
(lambda (x) (/ 1.0 (* x (+ x 2)))) returns a procedure that is like pi-term

(define (plus4 x) (+ x 4)) is equivalent to (define plus4 (lambda (x) (+ x 4)))

<u>**Note**</u>: Evaluating a lambda expression returns a **procedure object**

<u>**Therefore:**</u>

```
((lambda (x) (+ 1 x)) 2) => 3
```

in the same way:

```
(define (increment x)
      (+ 1 x))
(increment 2) => 3
```

or

```
(define increment (lambda (x) (+ 1 x)))
(increment 2) => 3
```

Using Lambda to create local variables

Suppose

f(x,y) = x(1 + xy)(1 + xy) + y(1 -y) + (1 + xy)(1 -y)

if a = 1 + xy and b= 1-y

f(x,y) = xaa + yb + ab

Let's write the function f in scheme

```
(define (f x y)
   (define (f-helper a b)
      (+    (* x (square a))
            (* y b)
            (* a b)))
   (f-helper (+ 1 (* x  y))
               (- 1  y)))
```

We could use lambda instead creating two variables a b that get bound to (+ 1 (* x y)) and (- 1 y)) respectively

```
(define (f x y)
   ((lambda (a b)
         (+    (* x (square a))
               (* y b)
               (* a b)))
      (+ 1 (* x  y))
      (- 1  y)))
```

Special form Let: Syntactic sugar for Using Lambda to Create Local Variables

The syntax for lambda

```
((lambda (<var1> ...<varn>)
   <body>)
<exp1>
<exp2>
.
.
.
<expn>)
```

can be expressed using the special form let

```
(let ((<var1> <exp1>)
      (<var2> <exp2>)
      .
      .
      .
      (<varn> <expn>))
   <body>)
```

which can be thought of as

let <var1> have the value <exp1> and
<var2> have the value <exp2> and
.
.
.
<varn> have the value <expn> and
in <body>

Therefore

```
(let ((x 3)
      (y (+ 1 3)))
   (* x y))
```

=> will generate the value 12 when evaluated

```
(let ((x 3)
      (y (+ x 3)))
   (* x y))
```

=> will generate ? when evaluated

```
(let ((x 3))
     (let ((y (+ x 1))) (* x y)))
```

Alternatively, use let* to do this automatically

<u>Note:</u> Scope of variables is in let expression
Therefore:

```
(define x 5)
(+    (let (( x 3))
         (+ x (* x 10)))
      x)

=> 38
```

Back to our Example

```
(define (f x y)
   (define a (+ 1 (* x y)))
   (define b (- 1 y))
   (+    (* x (square a))
         ( * y b)
         (* a b))))
```

- You should save define for binding procedures to a name
- When you need temporary variable bindings, use let

```
(define (f x y)
   (let (( a (+ 1 (* x y)))
         ( b (- 1 y)))
      (+    (* x (square a))
            ( * y b)
            (* a b))))
```

Quiz

Suppose you have the following function f

```
(define (f g)
   (g 2))
```

(f square) =>?   4 is the answer

(f (lambda (z) (* z (+ z 1)))) =>? 6 is the answer

(f f) => ? we will get an error by attempting to evaluate 2 as a procedure, the trace is as follows

(f f)

```
(f 2)
(2 2)
```

Procedures as Return Values

Average-damp is a procedure that takes as it argument a procedure that returns as its value a procedure

```
(define (average-damp f)
   (lambda (x) (average x (f x))))
```

Applying average-damp to the square procedure returns a procedure whose value when evaluated is the average of x and x squared

```
((average-damp square) 10) => 55
```

3.5 Summary

- Procedures
  - named by variables
  - passed as arguments to procedures
  - returns as results from procedures
  - procedures can be created with lambda
- lambda can be used to create local variables
- let is syntactic sugar for lambda